Axiomatic Foundations for Metrics of Distributive Justice Shown by the Example of Needs-Based Justice

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Distributive justice deals with allocations of goods and bads within a group. Different principles and results of distributions are seen as possible ideals. Often those normative approaches are solely framed verbally, which complicates the application to different concrete distribution situations that are supposed to be evaluated in regard to justice. One possibility in order to frame this precisely and to allow for a fine-grained evaluation of justice lies in formal modelling of these ideals by metrics. Choosing a metric that is supposed to map a certain ideal has to be justified. Such justification might be given by demanding specific substantiated axioms, which have to be met by a metric. This paper introduces such axioms for metrics of distributive justice shown by the example of needs-based justice. Furthermore, some exemplary metrics of needs-based justice and a three dimensional method for visualisation of non-comparative justice axioms or evaluations are presented. Therewith, a base worth discussing for the evaluation and modelling of metrics of distributive justice is given.

Keywords: justice, distributive justice, needs-based justice, axiomatics, axioms, metrics, measures, indices, formal modelling.

Introduction[1]

Issues of distributive justice are omnipresent. Economics and politics as well as medicine or private citizens are faced by them. The difficulty of how to distribute a given good or bad has troubled thinkers for generations and led them to many very distinct normative theories. As a general rule those theories have in common that everybody should get a person's due. Disagreement seems to be centred upon how to legitimize such a person's due. Amongst other criteria in the debate are equality, equity, power and desert. Another criterion considered to be possible is that of need (cf. Forsyth, 2006).

Besides this problem of disagreement that encompasses the whole historic discourse, there is a certain inaccuracy that can be discussed: Thus far being generally formulated only verbally, it is not always clear how the different ideals of justice should be applied to different concrete distribution situations that are supposed to be evaluated in regard to justice. Often it is not possible to state which impact small variations in distributions are supposed to have on their justice evaluation.

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This inaccuracy might be dissolved if the ideals in question are modelled formally by metrics of distributive justice and thereby gaining precise mathematical auxiliaries that can achieve the evaluation of different distributive situations with regard to their distributive justice.[2] In order to prevent arbitrariness, the selection of such a metric can be justified by the demand of certain substantiated axioms.

Apart from a rudimentary attempt by Miller (cf. 1999) there are as yet no such metrics of needs-based justice. Jasso (cf. 1999, 2007, Jasso and Wegener 1997) proposes general metrics of justice that are formally more developed than Miller's but nonetheless not primarily axiomatically motivated.[3] Jasso (cf. 1978) refers to a number of other proposals by Adams (cf. 1965), Berger and colleagues (cf. 1972), Homans (cf. 1974) as well as Walster and colleagues (cf. 1976). Eriksson discusses further rudimentary metrics with Jasso in view (cf. Eriksson, 2012).

A similar issue, where metrics are applied, can be found in the measurement of povert, where some approaches make use of subjective poverty lines (cf. Goedhart et al., 1977, Flink and van Praag, 1991), as well as in the measurement of inequality and welfare with heterogeneous needs, where amongst others Atkinson and Bourguignon (cf. 1987) must be mentioned, as well as Lambert and Ramos (cf. 2002, Chakravarty, 2009).

This variety of prominent highlights does not by far exhaust the range of possible approaches that can be utilized for metrics of needs-based justice. But already here it becomes clear that some criteria are necessary by which this amount of possible approaches can be assessed. Again, it seems obvious to follow the research of measuring poverty, where – as seen above – a partially similar, though not identical, issue is at hand. With Sen (cf. 1976) the formulation of desirable axioms that a metric should meet ideally, has found wide dissemination. The following considerations on needs-based justice originated based on this approach. They can be extrapolated from this exemplary framework and applied onto different other combinations of legitimate demands and actual allocations, so they can be seen as a basis for an axiomatic of distributive justice in general.

In the following there is no coherent or self-contained axiomatic to be presented that could be understood as a necessary or sufficient set of consistent axioms for the assessment or the modelling of metrics of needs-based justice. Such an axiomatic would have to be justified first of all on normative grounds, which is not intended here. The intention is rather to analyse and present possibilities. This happens against the background of a separation of normative statements and scientific analysis, as pointed out by Max Weber.[4]

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[2] This may evoke associations with philosophers from the 17th century, when some philosophers orientated themselves towards mathematics. Exemplarily Spinoza comes into view, who developed his "Ethics" based on the "geometric method" with definitions, axioms, theorems and proofs.

[3] One exception might be given by the considerations regarding an axiom of comparison (cf. Jasso, 1990).

[4] Contrary perspectives have been discussed in the course of the German value judgement dispute (cf. Albert 2010) and in the course of the positivism dispute (cf. Dahms, 1994). Weber himself writes in the mentioned lecture: "For opinions on issues of practical politics and the academic analysis of political institutions and party policies are two very different things. If you speak about democracy at a public meeting there is no need to make a secret of your personal point of view. On the contrary, you have to take one side or the other explicitly; that is your damned duty. The words you use are not the tools of academic analysis, but a way of winning others over to your political point of view. They are not plowshares to loosen the solid soil of

He cites Tolstoy in his lecture on "Science as a vocation" on the questions, what the meaning of science could be: "Science is meaningless because it has no answer to the only questions that matter to us: 'What should we do? How shall we live?'" (Weber, 2004, p. 17) According to Weber, science could – relating to problems of practical philosophy – nonetheless provide clarity:

> Always assuming that clarity is something we ourselves possess. Insofar as we do, we can make clear to you that in practice we can adopt this or that attitude toward the value problem at issue [...]. Of course, he can say to you that if you wish to achieve this or that end, you will have to put up with certain accompanying consequences that experience tells us are bound to make their appearance. [...] This brings us to the last contribution that science can make in the service of clarity, and at the same time we reach its limits. We can and should tell you that the meaning of this or that practical stance can be inferred consistently, and hence also honestly, from this or that ultimate fundamental ideological position. It may be deducible from one position, or from a number – but there are other quite specific philosophies from which it cannot be inferred. [...] The discipline of philosophy and the discussion of what are ultimately the philosophical bases of the individual disciplines all attempt to achieve this. (Weber, 2004, p. 26)

It is in this sense that some first considerations on fundamental desiderata of normative claims are to be presented. However, this can not provide completeness. The presented desiderata are to be understood as axioms that are not meant to be individually necessary or collectively sufficient but as an arguable and expandable groundwork that can be useful for the compilation of consistent axiomatics.

1 Some first considerations on Axioms for Metrics of Distributive Justice shown by the Example of Needs-Based Justice

Using the example of needs-based justice, a first cataloguing of possible desiderata or axioms for metrics of distributive justice is to be introduced. What is not to be presented is a concrete axiomatic in the form of a selection of a consistent set out of this catalogue. This remains the task of a seperate normative discourse.

The need of an individual is regarded to be a fundamental factor for the evaluation of justice. How much an individual person should rightfully receive then depends on this very factor. By exchanging this basis of legitimation, an axiomatic could be extended to other principles of distributive justice.

To obtain a precise formalisation of the axioms first of all a notation is to be introduced. Subsequently, there are different classes of axioms – that is for measurement, monotonicity, transfers, population growth and sensitivity – to be presented that are followed by some final remarks.

contemplative thought, but swords to be used against your opponents: weapons, in short. In a lecture room it would be an outrage to make use of language in this way." (Weber, 2004, p. 20)

1.1 Notation[5]

A notation for the formalisation of the following axioms has to include the aspects that are considered to be relevant for metrics of needs-based justice. Such a selection of relevant aspects is, of course, never free of assumptions and always represents a more or less aware selection.

The individuals, whose needs and actual allocations of a given good or bad are to be considered in regards to a metric, are referred to as a set $\mathcal{P}$, consisting of n individuals $i = \{1, 2, ..., n\}$. Those individuals do not have to represent single persons. They can include groups of persons as well, for example households or institutions.

It is assumed that every individual i has an actual allocation γ_i of the given good or bad. This is quantified within the non-negative real numbers, $\gamma_i \in \mathbb{R}_{0+}$. Moreover let $\vec{\gamma} = \{\gamma_1, \gamma_2, ..., \gamma_n\}$ be a vector and

$$\Gamma = \sum_{i=1}^{n} \gamma_i$$

be the sum of the overall available amount of the given good or bad that does not have to be limited to physical goods, but that of course has to be quantifiable.

With regard to the given good or bad whose distribution is to be assessed, it is assumed that every individual i has, independent from its γ_i, a need that is denoted ν_i and that is used to determine, when the individual is considered as undersupplied, supplied or oversupplied with respect to that given good or bad. It also is quantified within the non-negative real numbers, $\nu_i \in \mathbb{R}_{0+}$. Here let $\vec{\nu} = \{\nu_1, \nu_2, ..., \nu_n\}$ be a vector and

$$N = \sum_{i=1}^{n} \nu_i$$

be the sum of the overall existing needs for the given good or bad. Using γ_i and ν_i, one can now distinguish an individual i in terms of its supply situation. From this classification we obtain the subsets $\mathcal{U}$, $\mathcal{S}$ and $\mathcal{O}$ from the set $\mathcal{P}$.[6]

An individual is considered as undersupplied with respect to a given good or bad, when it has less units of it than its need demands. It is considered as supplied, if it has exactly as many units as its need demands. When it has more units of it than its need demands, it is considered as oversupplied.

Definition 1 (Undersupply): i is undersupplied, if $\gamma_i < \nu_i$; the set of undersupplied individuals is $\mathcal{U} = \{i \in \mathcal{P} : \gamma_i < \nu_i\}$.

Definition 2 (Supply): i is supplied, if $\gamma_i = \nu_i$; the set of supplied individuals is $\mathcal{S} = \{i \in \mathcal{P} : \gamma_i = \nu_i\}$.

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[5] The presented notation is apart from some minor deviations a result of joint work with Mark Siebel, Nils Springhorn, Stefan Traub and Arne Robert Weiß within the subproject “Metrics of needs-based justice, expertise and coherence” as part of the research group “Needs-based justice and distribution procedures” funded by the German Research Foundation.

[6] Gamma and Ny have been chosen following the terms “goods” and “needs”. The denotions of the subsets follow the terms “undersupplied”, “supplied” and “oversupplied”.

DEFINITION 3 (OVERSUPPLY): i is oversupplied, if $\gamma_i > \nu_i$; the set of oversupplied individuals is
$\mathcal{O} = \{i \in \mathcal{P} : \gamma_i > \nu_i\}$.

Following the distinction of micro and macro justice, this perspective on individual allotments has to be aggregated to indices that focus on the overall justice of a distribution (cf. Brickman et al., 1981, Berger et al., 1972, Arts et al., 1991, Jasso, 1983). Such indices are denoted J.

DEFINITION 4 (INDEX OF DISTRIBUTIVE JUSTICE): An index of distributive justice is a function $J: \mathbb{R}^n x \mathbb{R}^n \rightarrow \mathbb{R}$.

To compare the indicated justice that an index attributes ti several distributions at least on an ordinal level, it is defined that an index should present a lower function value for a distribution that is considered as more just.

DEFINITION 5 (ORDER OF DISTRIBUTIVE JUSTICE VALUES): An index of distributive justice J denotes higher justice to a distribution $(\overrightarrow{\gamma_a}, \overrightarrow{\nu_a})$ than it does to a distribution $(\overrightarrow{\gamma_b}, \overrightarrow{\nu_b})$ if

$$J(\overrightarrow{\gamma_a}, \overrightarrow{\nu_a}) < J(\overrightarrow{\gamma_b}, \overrightarrow{\nu_b}).$$

While this definition only includes an interpretation of the function value, it is up to the following proposals for axioms to determine, which alternative of two distributions is to be regarded as more or less just.

1.2 Exemplary Metrics of Needs-Based Justice

To illustrate how such indices of needs-based justice might look like, some exemplary ones shall be introduced at this point. A consistent axiomatic that can include parts of the following suggestion, can then be used to asses those or other indices.

One obvious starting point for metrics of needs-based justice might be the justice evaluation function or the justice indices presented by Jasso (cf. 1978, 1980, 1990, 1996, 1999, 2007, Jasso und Wegener, 1997). For Jasso, justice evaluations are based on a comparison of an actual allotment and a person's due, which is not ideologically specified by her. For this comparison, she uses the natural logarithm of the ratio of the two variables. Adopted to the above introduced notation it can be written as:

$$J_{\text{Jasso}}(\gamma_i, \nu_i) = ln\left(\frac{\gamma_i}{\nu_i}\right)$$

Jasso – and Watts below – follow another interpretation of the function value than the one demanded by definition 5. In her case it is possible to interpret a functional value of 0 as a situation of being exactly supplied, while negative values represent an unjust undersupply and positive values represent an unjust oversupply; with absolute values having a stronger impact for cases of undersupply through the logarithmic function.

Jasso (cf. 1999) suggests the arithmetic mean as a possible way of aggregating such individual justice evaluation functions. A possible aggregation of such individual justice evaluations that has similarities to the poverty index of Watts (cf. 1968, Zheng, 1993) could be written as follows:

$$J_{\text{Watts}}(\vec{\gamma}, \vec{\nu}) = \frac{1}{n}\sum_{i=1}^{n} ln\left(\frac{\gamma_i}{\nu_i}\right)$$

After pointing out the closeness to the measurement of poverty, there is another option offered by the transformation of the relative poverty gap or the poverty measure from Foster and colleagues (cf. 1984) respectively, which is quite prominent in the field of poverty measurement (cf. Kockläuner, 2012). Transferred to the above introduced notation, the measure of Foster and colleagues can then be written as follows:

$$J_{\text{Foster}}(\vec{\gamma}, \vec{\nu}) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\vartheta - \gamma_i)}{\vartheta}\right)^{\alpha}$$

In the same way as the relative poverty gap, here the ratios of the differences from poverty line – denoted as ϑ – and the actual allotments are in view. Foster extends this with the power of α that can be interpreted as a constant proportional risk aversion (cf. Kockläuner, 2012).

In this form, the index could be transferred to consider undersupplied individuals in means of needs. To achieve an index that considers the whole set of individuals it could be extended as follows, where, through U and O, the subsets of the under- and oversupplied individuals are taken into account separately, while P takes the set of individuals as a whole into consideration:

$$J_{\text{bauer}}(\vec{\gamma}, \vec{\nu}) = \frac{|U|}{|P|}\sum_{i \in U}\left(\frac{(\nu_i - \gamma_i)}{\nu_i}\right)^{\alpha} + \frac{|O|}{|P|}\sum_{i \in O}\left(\frac{(\gamma_i - \nu_i)}{\gamma_i}\right)^{\beta}$$

Here the under- and oversupplied individuals are taken into account separately with two different sums, weighted by their share of the whole population. The powers α and β are parameters of aversion against under- or oversupply that can represent higher or lower aversions in accordance with their size; the higher the power, the higher the depicted injustice of the respective sum. With a value lower than 1, it would furthermore be possible to take affinities in account, for example for oversupply.

1.3 Possible Axioms of Needs-Based Justice

Besides those exemplary metrics it is in principle possible to use any function of needs and allotments as an index of needs-based justice. The resulting amount of possibilities has to be restricted in a reasonable way. This can be achieved – as stated above – by demanding certain axioms that such an index ideally has to meet (cf. Scheicher, 2009). In the following, axioms shall be understood as the stating of formal or content-wise substantiated properties that metric demands and that can constitute a basis for classifying metrics in those that are acceptable or those that have to be dismissed (cf. von der Lippe, 1996). Axioms or desiderata in this sense are used as a basis for constructing or evaluating metrics in different areas of research; for example, within the frame of economic sciences with regard to measurement of inequality, poverty or wealth (cf. Scheicher, 2009, Herlyn, 2012).

Amongst a variety of formally or methodically motivated axioms, which are to be presented at the beginning of the subsection, a number of content-wise motivated ones has to be introduced. In the following, some possible axioms are presented. They are meant to be a collection of discussable basic assumptions that can be composed modularly to assemble an axiomatic as a basis for the construction or evaluation of possible metrics. For better overview the axioms are divided into several classes: First the methodically motivated axioms are presented, followed by those of monotonicity, which deal with variations in allotments of a given good for a single individual. These axioms are followed by axioms of transfer, which focus on transfers between several individuals. Axioms of population growth deal with variations of a given population. Finally, axioms of sensitivity focus on the intensity of deviations from an allotment of an individual to its due need.

Thus, it shall be attempted to cover possible desirable properties of metrics within the scope of some obvious normative claims in the contexts of monotonicity, transfers, population growth and sensitivities that have been transformed and adjusted from poverty measurement. Thereby a catalogue of different conceivable axioms is presented; the axioms being in part mutually exclusive and thereby making a selection that has to orientate itself on the desired normative aspects that a certain index should represent, indispensable. This selection has to take place within the context of a normative discourse and has to face the thereby accompanying challenges and problems.

Methodically Motivated Axioms

The methodically motivated axioms sum up all those axioms that are not primarily motivated by normative reflections, but rather seem useful within the methodically scope of measuring. Nonetheless, they can of course have relevance for normative claims and should not be misunderstood as independent and without assumptions or as somehow neutral.

An axiom of scale invariance (cf. Seidl, 1988, Kockläuner, 2012) demands of indices that they do not change when the needs and allotments are scaled by the same factor. Analogous to metrics of poverty, it is possible to speak about relative metrics of needs-based justice when this axiom is satisfied.[7]

Axiom 1 (Scale invariance): $J(\lambda \cdot \vec{\gamma}, \lambda \cdot \vec{\nu}) = J(\vec{\gamma}, \vec{\nu}), \lambda > 0.$

Every index that is scale invariant in such a way also satisfies a demand for replication invariance: To allow for comparison of two groups $\mathcal{P}_a$ and $\mathcal{P}_b$ with different numbers of individuals, this axiom requires for poverty measurement that the function value of an index does not vary through a replication of individuals: Then one has $J(\overrightarrow{\gamma_a}, \overrightarrow{\nu_a}) = J(\overrightarrow{\gamma_b}, \overrightarrow{\nu_b})$ if $\overrightarrow{\gamma_b}$ and $\overrightarrow{\nu_b}$ are obtained from $\overrightarrow{\gamma_a}$ and $\overrightarrow{\nu_a}$ by depicting a replication, in which for every γ_i and ν_i out of $\overrightarrow{\gamma_a}$ and $\overrightarrow{\nu_a}$ there is a quantity increased by the factor of λ in $\overrightarrow{\gamma_b}$ and $\overrightarrow{\nu_b}$. Scale invariant indices also satisfy the demand for unity consistence that requires for indices to be independent from conversion in other unities, for example from Euro to Dollar (cf. Zheng, 2007).

Furthermore, an axiom of symmetry requires irrelevance concerning the question, which individual is associated with a certain pair γ_i, ν_i; a commutation of pairs between individuals should leave an index unchanged. This axiom makes sure that an evaluation of needs-based justice depends solely on the chosen factors – in this case the components of the pairs – and thereby is without distinction of person. In poverty measurement, such an

[7] Whereas a relative index therefore is invariant for same percentage changes, an absolute index is invariant for same changes in addition or subtraction: An according axiom of translation invariance can demand that the functional value of J does not vary if the needs and allotments are increased or reduced by the same δ.

axiom is strikingly named an axiom of anonymity; symbolically represented by the blindfold of Justitia.

AXIOM 2 (SYMMETRY): If $\overrightarrow{\nu_b}$ and $\overrightarrow{\gamma_b}$ are obtained from $\overrightarrow{\nu_a}$ and $\overrightarrow{\gamma_a}$ by changing two individuals i, j ∈ $\mathcal{P}$, then $J(\overrightarrow{\gamma_a}, \overrightarrow{\nu_a}) = J(\overrightarrow{\gamma_b}, \overrightarrow{\nu_b})$.

To avoid abrupt changes in the functional value of an index and therefor to make sure that infinitesimal changes inγandνonly lead to accordingly small changes in the functional value, an axiom of continuity can be demanded.

AXIOM 3 (CONTINUITY): $J(\vec{\gamma}, \vec{\nu})$ is continuous in γ and ν.

Since there is no concrete measurement unit for justice in general or needs-based justice in particular, like meter for length or bar for pressure, it is demanded that the index number should be a dimensionless value. With an axiom of normalization, it can be demanded that the value is normalized for example on the interval [0,1].

AXIOM 4 (NORMALIZATION): For all $\vec{\gamma}$ and $\vec{\nu}$ it holds that $0 \leq J(\vec{\gamma}, \vec{\nu}) \leq 1$.

The functional value of an index can be varied depending on preferred aspects of interpretation. Jasso for example uses a scale that is not constrained. In her case 0 represents a just allotment, while negative values represent an unjust shortage and positive values represent an unjust surplus of a given good or bad, with no maximum values in both cases (cf. Jasso, 2007). In the following a functional value of 0 shall depict a just allotment, while greater positive values represent unjust allotments. Only exception shall be the axiom of continuously increasing monotonicity.

Both the axiom of continuity and of normalization are important for the comparability of different sets $\mathcal{P}_a$ to $\mathcal{P}_m$ with regard to their needs-based justice. They are also relevant for the aspired scale level of an index that determines if several sets can only be interpreted on an ordinal scale or can be compared on a more fine grained scale, for example on an interval scale (cf. Stevens, 1947).

By forming subgroups of a set $\mathcal{P}$ along certain characteristics – for example age, social stratum or geographic location – it becomes possible to make statements on their share of the overall injustice. Those can give indications for reducing specific injustices for example through policy actions.[8] An axiom of subgroup consistency demands accordingly that the indicated needs-based justice of an index increases or decreases, if it does so in one of several disjunctive subgroups, while remaining constant in the others (cf. Foster et al., 1984, Foster and Shorrocks, 1991, Zheng, 1997).

AXIOM 5 (SUBGROUP CONSISTENCY): If $\overrightarrow{\gamma_a}$ and $\overrightarrow{\nu_a}$ are disaggregated into $\overrightarrow{\gamma_{a'}}$ and $\overrightarrow{\nu_{a'}}$ plus $\overrightarrow{\gamma_{a''}}$ and $\overrightarrow{\nu_{a''}}$ as well as $\overrightarrow{\gamma_b}$ and $\overrightarrow{\nu_b}$ are disaggregated into $\overrightarrow{\gamma_{b'}}$ and $\overrightarrow{\nu_{b'}}$ plus $\overrightarrow{\gamma_{b''}}$ and $\overrightarrow{\nu_{b''}}$ (with $\overrightarrow{\gamma_a} =$

[8] In the context of measuring poverty, Anand has early made the importance of decomposability clear. Referring to the above mentioned Sen, he states: „The Sen measure is, unfortunately, not decomposable between groups. Yet, in the design of poverty redressal policies, it would seem important to be informed of the extent to which a particular group accounts for overall poverty. [...] A diagnosis of poverty requires answers to questions such as: Who are the poor? Where are they located? In which sectors do they work? What are the characteristics of the poor that are different from those of the non-poor?" (Anand, 1977, p. 12)

$\overrightarrow{\gamma_{a'}} + \overrightarrow{\gamma_{a''}}, \overrightarrow{\nu_a} = \overrightarrow{\nu_{a'}} + \overrightarrow{\nu_{a''}}$ and $\overrightarrow{\gamma_b} = \overrightarrow{\gamma_{b'}} + \overrightarrow{\gamma_{b''}}, \overrightarrow{\nu_b} = \overrightarrow{\nu_{b'}} + \overrightarrow{\nu_{b''}})$ with $J(\overrightarrow{\gamma_{a'}}, \overrightarrow{\nu_{a'}}) < J(\overrightarrow{\gamma_{b'}}, \overrightarrow{\nu_{b'}})$ and $J(\overrightarrow{\gamma_{a''}}, \overrightarrow{\nu_{a''}}) = J(\overrightarrow{\gamma_{b''}}, \overrightarrow{\nu_{b''}})$, then $J(\overrightarrow{\gamma_a}, \overrightarrow{\nu_a}) < J(\overrightarrow{\gamma_b}, \overrightarrow{\nu_b})$.

An axiom of weighted decomposability eventually demands that an index of an evaluated set $\mathcal{P}$ should be equal to the sum of its proportionally weighted subsets $\mathcal{P}_a$ to $\mathcal{P}_m$, if M subgroups m = {1, 2, …, m} are composed with a population share of n_z / n each, so that an increase or decrease in one of the subgroups' index has an effect according to the number of individuals the subgroup contains.

Axiom 6 (Weighted Decomposability): It holds $J(\vec{\gamma}, \vec{\nu}) = \sum_{m=1}^{M} \frac{n_z}{n} J(\overrightarrow{\gamma_z}, \overrightarrow{\nu_z})$.

Axioms of Monotonicity

Axioms of monotonicity consider changes in the allotment γ_i of an individual i. They are non-comparative, that is, they rely only on the parameters of need and allotment and do not depend on factors like the position of an individual with regard to the remaining individuals of a group, as it could be demanded from a comparative point of view (cf. Feinberg, 1974, Montague, 1980).

Based on the mathematical description of functions it is possible to describe several cases for changes in the allotment γ_i and the associated values of justice through the term of monotonicity. Justice is monotonic increasing if the function value of J does not denote lower justice with an increase of γ_i, so according to definition 5 the function value of an index J does not increase.

Definition 6 (Monotonic Increasing Justice): If $\gamma_i < \gamma_i'$ then $J(\nu_i, \gamma_i) \geq J(\nu_i, \gamma_i')$.

Justice is called strictly monotonic increasing if the function value of J does denote greater justice with an increase of γ_i.

Definition 7 (Strictly Monotonic Increasing Justice): If $\gamma_i < \gamma_i'$ then $J(\nu_i, \gamma_i) > (\nu_i, \gamma_i')$.

It is instead called monotonic decreasing if the function value of J does not denote greater justice with an increase of γ_i.

Definition 8 (Monotonic Decreasing Justice): If $\gamma_i < \gamma_i'$ then $J(\nu_i, \gamma_i) \leq (\nu_i, \gamma_i')$.

Finally, justice can be called strictly monotonic decreasing if the function value of J does denote lower justice with an increase of γ_i.

Definition 9 (Strictly Monotonic Decreasing Justice): If $\gamma_i < \gamma_i'$ then $J(\nu_i, \gamma_i) < J(\nu_i, \gamma_i')$.

An index J can of course be sectionwise defined with regards to its monotonicity. Furthermore, it is possible that different kinds of needs evoke different definitions of monotonicity. While in the following the value of allotment γ_i is varied exclusively, it is of course also conceivable to vary the value of need ν_i, which should be assumed to be static in this paper, so that it is possible to concentrate on distributions of goods or bads with heterogeneous but unchanging needs. Moreover, there are two possible ways of formulating axioms; either the ratio or the absolute distance of needs and allotments is considered. While this may appear within the scope of axioms of monotonicity as merely two different representation methods, since both will lead the axioms to identical conclusions, they obtain importance for the following axioms of transfer: Here it becomes possible to have identical distributions that lead the axioms in their absolute or relative formulation to demanding different behaviours for an index. For that reason, some of the axioms shall be presented in both ways.

The idea of monotonicity seems simple for cases of undersupply: As long as an individual has a lack of a certain good, an increase of its allotment should be understood as more just. But for the case of oversupply, there are different normative claims imaginable.

Axioms of invert monotonicity can be associated with the principle of homoeostasis, that – exemplarily for physiological contexts – assumes that things level off between states of too much and too little. One may also think about the concept of μεσότης in the work of Aristotle, who makes the disposition between lack and excess central for his conception of virtue, or about the philosophical debate on theories of sufficiency. Moreover, invert monotonicity has special relevance for the possibility of meeting the needs of undersupplied individuals: That what constitutes the oversupply of one person could be used to meet the need of another person that is undersupplied with regards to the considered good – at least if other principles of justice are suppressed so that only needs are in focus.

For the first possible axiom of monotonicity it shall be assumed that an exact meeting of needs is the ideal state for a distribution, so that both undersupply and oversupply are perceived as unjust. This does not mean that an identical absolute value of under- and oversupply has to be treated identical as will be seen with the axioms of sensitivity.

The axiom of invert monotonicity in its relative form demands accordingly that – ceteris paribus – a change of the allotment γ_i of an individual i out of the set $\mathcal{P}$ lets denote an index J greater needs-based justice, if the ratio of allotment γ_i and need ν_i is smaller afterwards. Thereby it is assumed – as discussed within the scope of an axiom of normalization – that greater needs-based justice is indicated by a lower value of J towards 0.

AXIOM 7.1 (RELATIVE INVERT MONOTONICITY): If $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i $\in \mathcal{P}$ a δ so that $\gamma_{bi} = \gamma_{ai} \pm \delta$, then $J(\overrightarrow{\gamma_a}, \vec{\nu}) > J(\overrightarrow{\gamma_b}, \vec{\nu})$,if $P_{ai} > P_{bi}$, or then $J(\overrightarrow{\gamma_a}, \vec{\nu}) < J(\overrightarrow{\gamma_b}, \vec{\nu})$, if $P_{ai} < P_{bi}$ (with $P = \gamma / \nu$, if $(\nu / \gamma) < 1$, and $P = \nu / \gamma$, if $(\nu / \gamma) > 1$).

In the absolute variation this axiom considers no longer the ratio of allotment γ_i and need ν_i but the absolute distances between the two variables.

AXIOM 7.2 (ABSOLUTE INVERT MONOTONICITY): If $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i $\in$ P a δ so that $\gamma_{bi} = \gamma_{ai} \pm \delta$, then $J(\overrightarrow{\gamma_a}, \vec{\nu}) > J(\overrightarrow{\gamma_b}, \vec{\nu})$, if $|\nu_i - \gamma_{ai}| > |\nu_i - \gamma_{bi}|$, or then $J(\overrightarrow{\gamma_a}, \vec{\nu}) < J(\overrightarrow{\gamma_b}, \vec{\nu})$, if $|\nu_i - \gamma_{ai}| < |\nu_i - \gamma_{bi}|$.

Those requirements can be depicted in two and three dimensional visualisations. In the following examples the justice increases (the function value of J decreases) in a linear manner with a convergence of allotment and need. The function value is 0 if allotment and need meet.

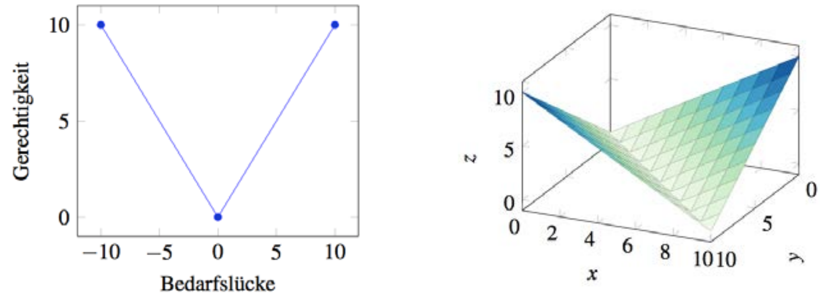


Fig. 1: Two and three dimensional visual examples for invert monotonicity of justice

For the three dimensional visualisation the function values for the possible pairs from ν_i and γ_i of some individual i are depicted with $\gamma_i = x$ and $\nu_i = y$ on an intervall [0, 10] with x, y ∈ ℕ, where J = z and exemplarily z = |x – y|.

With another variation of an axiom of monotonicity it would be possible to demand the same for undersupply and supply, while being different for the case of oversupply, so that changes of allotments above a needs threshold would not change the functional value. This could be desirable if one assumes that a concept of needs is only applicable for cases of undersupply. An axiom of limited monotonicity demands accordingly that – ceteris paribus – a change of the allotment γ_i of an individual i out of the set $\mathcal{P}$ lets denote an index J greater needs-based justice, if the relative distance between allotment γ_i and need ν_i is smaller afterwards, provided that neither initially nor finally a situation of being supplied or oversupplied is given. Otherwise the index does not change, if the allotment varies.

AXIOM 8.1 (RELATIVE LIMITED MONOTONICITY): If $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ so that $\gamma_{bi} = \gamma_{ai} \pm \delta$, then $J(\overrightarrow{\gamma_a}, \vec{v}) > J(\overrightarrow{\gamma_b}, \vec{v})$, if $P_{ai} > P_{bi}$, or then$J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$, if $P_{ai} < P_{bi}$ (with $P = \gamma / \nu$, if $(\nu / \gamma) < 1$, and $P = \nu / \gamma$, if $(\nu / \gamma) > 1$), provided that i in $\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b}$ $\notin \mathcal{S} \cup \mathcal{O}$. Provided that i in $\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b} \in \mathcal{S} \cup \mathcal{O}$, it holds $J(\overrightarrow{\gamma_a}, \vec{v}) = J(\overrightarrow{\gamma_b}, \vec{v})$.

AXIOM 8.2 (ABSOLUTE LIMITED MONOTONICITY): If $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ P a δ so that $\gamma_{bi} = \gamma_{ai} \pm \delta$, then $J(\overrightarrow{\gamma_a}, \vec{v}) > J(\overrightarrow{\gamma_b}, \vec{v})$, if $|\nu_i - \gamma_{ai}| > |\nu_i - \gamma_{bi}|$, or then $J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$, if $|\nu_i - \gamma_{ai}| < |\nu_i - \gamma_{bi}|$, provided that i in$\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b} \notin \mathcal{S} \cup \mathcal{O}$. Provided that i in $\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b} \in \mathcal{S} \cup \mathcal{O}$, it holds $J(\overrightarrow{\gamma_a}, \vec{v}) = J(\overrightarrow{\gamma_b}, \vec{v})$.

This demands can also be presented in two and three dimensional visualisations, where the function value only increases in cases of undersupply. In cases of oversupply it remains constant.

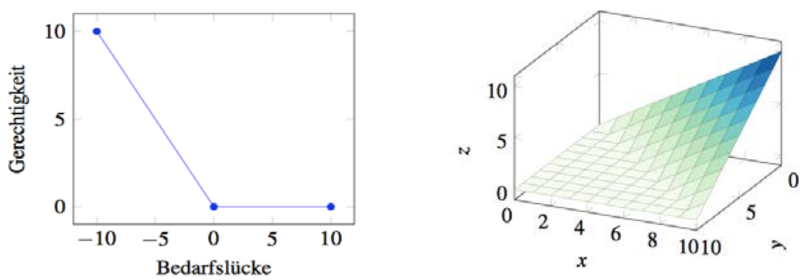


Fig. 2: Two and three dimensional visual examples for limited monotonicity of justice

With a third variation of an axiom of monotonicity it would be possible to demand independently from the situation of supply that an increase in allotment lets an index indicate a greater needs-based justice. An axiom of continuously increasing monotonicity demands accordingly that – ceteris paribus – a change of the allotment γ_i of an individual i out of the set $\mathcal{P}$ lets denote an index J greater needs-based justice, if the relative distance between allotment γ_i and need ν_i is smaller afterwards in the range of undersupply or greater afterwards in the range of oversupply.

AXIOM 9.1 (RELATIVE CONTINUOUSLY INCREASING MONOTONICITY): If $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ so that $\gamma_{bi} = \gamma_{ai} \pm \delta$, then $J(\overrightarrow{\gamma_a}, \vec{v}) > J(\overrightarrow{\gamma_b}, \vec{v})$, if $P_{ai} > P_{bi}$, or then $J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$, if $P_{ai} < P_{bi}$ (with $P = \gamma / \nu$, if $(\nu / \gamma) < 1$, and $P = \nu / \gamma$, if $(\nu / \gamma) > 1$), provided that i in $\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b} \in \mathcal{U}, \mathcal{S}$. Provided that i in $\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b} \in \mathcal{S} \cup \mathcal{O}$, it holds $J(\overrightarrow{\gamma_a}, \vec{v}) > J(\overrightarrow{\gamma_b}, \vec{v})$, if $P_{ai} < P_b$, or then $J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$, if $P_{ai} > P_b$.

AXIOM 9.2 (ABSOLUTE CONTINUOUSLY INCREASING MONOTONICITY): If $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ P a δ so that $\gamma_{bi} = \gamma_{ai} \pm \delta$, then $J(\overrightarrow{\gamma_a}, \vec{v}) > J(\overrightarrow{\gamma_b}, \vec{v})$, if $|\nu_i - \gamma_{ai}| > |\nu_i - \gamma_{bi}|$, or then $J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$, if $|\nu_i - \gamma_{ai}| < |\nu_i - \gamma_{bi}|$, provided that i in $\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b} \in \mathcal{U}, \mathcal{S}$. Provided that i in $\overrightarrow{\gamma_a}$, i in $\overrightarrow{\gamma_b} \in \mathcal{S} \cup \mathcal{O}$, it holds $J(\overrightarrow{\gamma_a}, \vec{v}) > J(\overrightarrow{\gamma_b}, \vec{v})$, if $|\nu_i - \gamma_{ai}| < |\nu_i - \gamma_{bi}|$, or $J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$, if $|\nu_i - \gamma_{ai}| > |\nu_i - \gamma_{bi}|$.

On an increased interval these axioms can be visualised as follows, where the function value of J decreases with further oversupply and so depict greater justice.

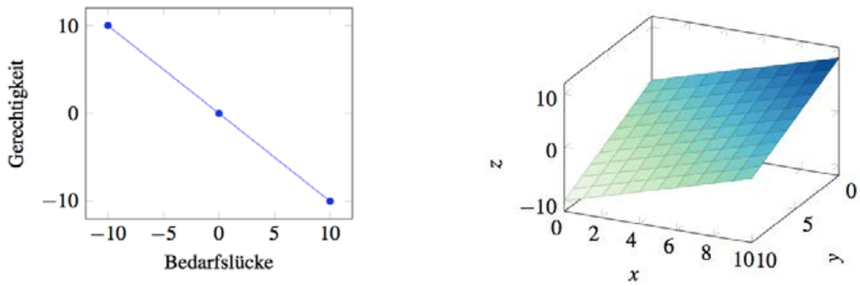


Fig. 3: Two and three dimensional visual examples for continuously increasing monotonicity of justice

Axioms of Transfer

Axioms of transfer deal with reallocations of a given quantity Γ of a good or bad between several individuals, here denoted as i and j.

DEFINITION 10 (TRANSFER): γ_i of i ∈ $\mathcal{P}$ is reduced by δ with $0 < \delta \le \gamma_i$ and γ_j of j ∈ $\mathcal{P}$ is extended by the same δ simultaniousyl.

The focus here is on the situation in which a distribution resulting of a transfer is regarded as more or less just than the initial situation. Against the background of the aboveintroduced axiom of monotonicity, this can at least be interpreted in three different ways.[9]

[9] Pigou and Dalton established considerations about transfers in the scope of welfare were established by Pigou and Dalton (cf. Dalton, 1920). Pigou writes: „My second proposition can be stated in several ways. The most abstract form of it affirms

Due to space restrictions, only axioms of transfer for invert monotonicity are to be introduced exemplarily. In this case the more just distribution is that, which either has a smaller absolute distance between allotment and need or has a smaller ratio.

Those two possibilities apply for all presented content-wise motivated axioms. Either the ratio or the absolute distance of needs and allotments is considered. While this may appear within the scope of axioms of monotonicity as merely two different representation methods, since both will lead the axioms to identical conclusions, they obtain importance for the following axioms of transfer: Here it becomes possible to have identical distributions that lead the axioms in their absolute or relative formulation to demanding different behaviours for an index.

Due to space restrictions only the relative version is to be presented in the following. In the appendices A and B the content-wise motivated axioms are presented in their absolute and relative version for all three introduced variations of monotonicity.

In case of an individual i transferring a part of its allotment γ_i to an individual j, so that the sum of both ratios of needs and allotments that is denoted with the greek P, decreases, it can be spoken of a positive transfer. For a neutral transfer the ratios do not change in sum. For a negative transfer, the ratios rise in sum.

AXIOM 10.1 (RELATIVE POSITIVE TRANSFER FOR INVERT MONOTONICITY): If for a given $\vec{\nu}$ a $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ with 0 < δ ≤ γ_{ai}, that is transferred from γ_i to γ_j of a j ∈ $\mathcal{P}$, so that finally ($P_{ai} + P_{aj}$) > ($P_{bi} + P_{bj}$) (with P = γ / ν, if (ν / γ) < 1, and P = ν / γ, if (ν / γ) > 1), then $J(\overrightarrow{\gamma_a}, \vec{\nu}) > J(\overrightarrow{\gamma_b}, \vec{\nu})$.

AXIOM 10.2 (ABSOLUTE POSITIVE TRANSFER FOR INVERT MONOTONICITY): If for a given $\vec{\nu}$ a $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ with 0 < δ ≤ γ_{ai}, that is transferred from γ_i to γ_j of a j ∈ $\mathcal{P}$, so that finally ($|\nu_{ai} - \gamma_{ai}| + |\nu_{aj} - \gamma_{aj}|$) > ($|\nu_{bi} - \gamma_{bi}| + |\nu_{bj} - \gamma_{bj}|$), then $J(\overrightarrow{\gamma_a}, \vec{\nu}) > J(\overrightarrow{\gamma_b}, \vec{\nu})$.

AXIOM 11.1 (RELATIVE NEUTRAL TRANSFER FOR INVERT MONOTONICITY): If for a given $\vec{\nu}$ a $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ with 0 < δ ≤ γ_{ai}, that is transferred from γ_i to γ_j of a j ∈ $\mathcal{P}$, so that finally ($P_{ai} + P_{aj}$) = ($P_{bi} + P_{bj}$) (with P = γ / ν, if (ν / γ) < 1, and P = ν / γ, if (ν / γ) > 1), then $J(\overrightarrow{\gamma_a}, \vec{\nu}) = J(\overrightarrow{\gamma_b}, \vec{\nu})$.

AXIOM 11.2 (ABSOLUTE NEUTRAL TRANSFER FOR INVERTER MONOTONICITY): If for a given $\vec{\nu}$ a $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ with 0 < δ ≤ γ_{ai}, that is transferred from γ_i to γ_j of a j ∈ $\mathcal{P}$, so that finally ($|\nu_{ai} - \gamma_{ai}| + |\nu_{aj} - \gamma_{aj}|$) = ($|\nu_{bi} - \gamma_{bi}| + |\nu_{bj} - \gamma_{bj}|$), then $J(\overrightarrow{\gamma_a}, \vec{\nu}) = J(\overrightarrow{\gamma_b}, \vec{\nu})$.

AXIOM 12.1 (RELATIVE NEGATIVE TRANSFER FOR INVERT MONOTONICITY): If for a given $\vec{\nu}$ a $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ with 0 < δ ≤ γ_{ai}, that is transferred from

that economic welfare is likely to be augmented by anything that, leaving other things unaltered, renders the distribution of the national dividend less unequal. If we assume all members of the community to be of similar temperament, and if these members are only two in number, it is easily shown that any transference from the richer to the poorer of the two, since it enables more intense wants to be satisfied at the expense of less intense wants, must increase the aggregate sum of satisfaction." (Pigou, 1912, p. 24) Contrary to the relevant literature on metrics of poverty, there shall no preservation of ranks be demanded for the case of transfers. Such ranks could be established for example by ranking individuals in case of undersupply from the greatest distance between allotment and need to the smallest distance and for the case of oversupply by ranking further from the smallest distance to the greatest distance between allotment and need.

γ_i to γ_j of a j ∈ $\mathcal{P}$, so that finally ($P_{ai} + P_{aj}$) < ($P_{bi} + P_{bj}$) (with P = γ / ν, if (ν / γ) < 1, and P = ν / γ, if (ν / γ) > 1) then $J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$.

AXIOM 12.2 (ABSOLUTE NEGATIVE TRANSFER FOR INVERT MONOTONICITY): If for a given $\vec{v}$ a $\overrightarrow{\gamma_b}$ is retrieved from $\overrightarrow{\gamma_a}$ by having for an i ∈ $\mathcal{P}$ a δ with 0 < δ ≤ γ_{ai}, that is transferred from γ_i to γ_j of a j ∈ $\mathcal{P}$, so that finally ($|\nu_{ai} - \gamma_{ai}| + |\nu_{aj} - \gamma_{aj}|$) < ($|\nu_{bi} - \gamma_{bi}| + |\nu_{bj} - \gamma_{bj}|$), then $J(\overrightarrow{\gamma_a}, \vec{v}) < J(\overrightarrow{\gamma_b}, \vec{v})$.

Of course the absolute formalisations of those axioms are more interesting only for the remaining two variations of monotonicity, they remain somewhat trivial for invert monotonicity of justice.

Axioms of Populations Growth

The previous axioms were focused on fix populations of $\mathcal{P}$ in regard of changes in the allotments of a single individual or for several individuals in the case of transfers. With axioms of population growth now the behaviour of an index for changing population sizes is to be described.

An axiom of growth of under- or oversupplied population for invert monotonicity demands that – ceteris paribus – an index should depict greater injustice, if $\mathcal{P}$ is extended by an addition individual j that is under- or oversupplied. – In contrast it holds, that for continuously increasing monotonicity such a depiction of greater injustice would be demanded only for an additional undersupplied individual.

AXIOM 13 (GROWTH OF UNDER- OR OVERSUPPLIED POPULATION FOR INVERT MONOTONICITY): If $\overrightarrow{\gamma_b}$ and $\overrightarrow{v_b}$ are retrieved from $\overrightarrow{\gamma_a}$ and $\overrightarrow{v_a}$ by having for n individuals in $\mathcal{P}_a$ finally n + 1 in $\mathcal{P}_b$ with $\overrightarrow{\gamma_J} > \overrightarrow{v_J}$ or $\overrightarrow{\gamma_J} < \overrightarrow{v_J}$, for the additional j ∈ P, then $J(\overrightarrow{\gamma_a}, \overrightarrow{v_a}) < J(\overrightarrow{\gamma_b}, \overrightarrow{v_b})$.

An additional axiom of growth of supplied population for invert monotonicity demands that – ceteris paribus – an index should depict greater injustice, if $\mathcal{P}$ is extended by an additional individual j that is supplied.

AXIOM 14 (GROWTH OF SUPPLIED POPULATION FOR INVERT MONOTONICITY): If $\overrightarrow{\gamma_b}$ and $\overrightarrow{v_b}$ are retrieved from $\overrightarrow{\gamma_a}$ and $\overrightarrow{v_a}$ by having for n individuals in $\mathcal{P}_a$ finally n + 1 in $\mathcal{P}_b$ with $\overrightarrow{\gamma_J} = \overrightarrow{v_J}$, for the additional j ∈ $\mathcal{P}$, then $J(\overrightarrow{\gamma_a}, \overrightarrow{v_a}) > J(\overrightarrow{\gamma_b}, \overrightarrow{v_b})$.

Axioms of Sensitivity

Finally, axioms of sensitivity focus on the value of the under- or oversupply of an individual. Under and over a certain threshold of needs it is possible to apply several sensitivities. For example, an identical absolute value by which an allotment is decreased, weights heavier, if the starting point is farer away of the threshold of needs. An increase by the same absolute value above the threshold weights heavier if the starting point is closer to the threshold. This and other combinations can be depicted by the axioms for concave and convex sensitivity of monotonicity.

For this purpose, the demand of an axiom is: The greater the relative distance of need and allotment is, the deeper is the impact of the change by the same absolute value.

AXIOM 15 (CONCAVE SENSITIVITY OF MONOTONICITY): If for a given $\vec{v}$ a $\overrightarrow{\gamma_b}$ and $\overrightarrow{\gamma_c}$ are retrieved from $\overrightarrow{\gamma_a}$ by having an i ∈ $\mathcal{P}$ in $\overrightarrow{\gamma_b}$ and a j ∈ P in $\overrightarrow{\gamma_c}$ with initially

| $\nu_i - \gamma_i$ | < | $\nu_j - \gamma_j$ |, that are altered by the same δ with $\delta > 0$ that is subtracted if i, j ∈ $\mathcal{U}$ and added by i, j ∈ $\mathcal{O}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) \gtreqless J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$, depending on the kind of monotonicity that is assumed:

(1) if i, j ∈ $\mathcal{U}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) < J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$,

(2) if invert monotonicity is assumed and i, j ∈ $\mathcal{S} \cup \mathcal{O}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) < J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$,

(3) if continuously increasing monotonicity is assumed and i, j ∈ $\mathcal{S} \cup \mathcal{O}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) > J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$.

Accordingly, an axiom of convex sensitivity of monotonicity demands vice versa: The smaller the relative distance of need and allotment is, the deeper is the impact of the change by the same absolute value.

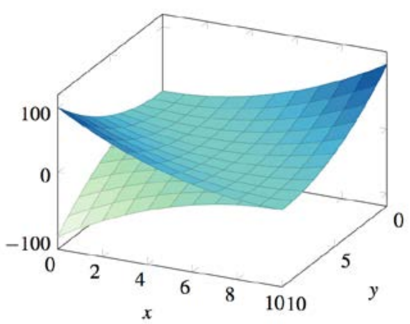
AXIOM 16 (CONVEX SENSITIVITY OF MONOTONICITY): If for a given $\vec{v}$ a $\overrightarrow{\gamma_b}$ and $\overrightarrow{\gamma_c}$ are retrieved from $\overrightarrow{\gamma_a}$ by having an i ∈ $\mathcal{P}$ in $\overrightarrow{\gamma_b}$ and a j ∈ P in $\overrightarrow{\gamma_c}$ with initially | $\nu_i - \gamma_i$ | < | $\nu_j - \gamma_j$ |, that are altered by the same δ with $\delta > 0$ that is subtracted if i, j ∈ $\mathcal{U}$ and added by i, j ∈ $\mathcal{O}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) \gtreqless J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$, depending on the kind of monotonicity that is assumed:

(1) if i, j ∈ $\mathcal{U}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) > J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$,

(2) if invert monotonicity is assumed and i, j ∈ $\mathcal{S} \cup \mathcal{O}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) > J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$,

(3) if continuously increasing monotonicity is assumed and i, j ∈ $\mathcal{S} \cup \mathcal{O}$, then $J(\overrightarrow{\gamma_b},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v}) < J(\overrightarrow{\gamma_c},\vec{v}) - J(\overrightarrow{\gamma_a},\vec{v})$.

Those sensitivities can also be visualised for the three kinds of monotonicity using the three dimensional model that has been applied above. It shows exemplarily the functional value or the justice value of J for all possible pairs of γ_i and ν_i from an individual i with γ_i = x and ν_i = y on an interval [−100, 100] with x, y ∈ Z and J = z. Depicted are exemplarily possible curve progressions for both concave and convex sensitivities for all three kinds of monotonicity.



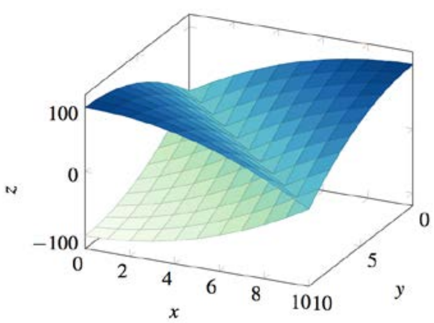


Fig. 4: Two and three dimensional visual examples for concave and convex sensitivities for all three kinds of monotonicity

2 Conclusion

Based on these axioms a variety of subsequent research projects could be justified. Of course, the presented axioms do not exhaust the possible normative assumptions; and still every single one of them is debatable. For example, the presented axioms are not orientated on aspects of procedural justice but focus on justice of outcome. In this regard an extension is conceivable, for example by including the criterion of Pareto efficiency. Moreover, the axioms are meant to be understood within the frame of non-comparative justice: The evaluation of needs-based justice relies on the ratio of needs and allotments of the individual. It could be considered to extend or replace the axioms by other principles of justice in order to get a comparative perspective and thereby for example focus on the position of an individual compared to other members of a group.

Moreover, the relation of the given axioms among themselves is interesting: Which axioms are compatible with each other? Is there a plausible set of maximal consistent axioms? Furthermore, it is obvious to ask how each axiom relates to concrete normative theories, and it is also obvious to ask how they can be used to form a basis for depicting empirical evaluations of justice. Can these axioms be useful for modelling the justice evaluations of laymen (cf. Schokkaert, 1999)?

Even though the axioms are formulated within the framework of needs-based justice, they are not limited to this very frame – at least not all of them. It is possible to transfer them to other ideals of justice. They also could be extended to more general metrics of justice with a plurality of concepts.

Finally, it is possible to use a justified and consistent subset of the presented axioms to evaluate possible indices or to form a basis for modelling such indices.

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